国立大学法人
鳴門教育大学
Naruto University of Education

Mathematics Education for Ensuring Children's Learning

Focusing on the "Understanding of Numbers", the "Consolidation of Learning Content", and "Textbook Structure"

Naruto University of Education
Satoshi Kusaka

(Naru★One)

Self-Introduction

Satoshi Kusaka (Ph.D.)

Associate Professor, Naruto University of Education (Mathematics Education)

Countries I Have Worked with in Education

South Sudan, Ethiopia, Ghana, Zambia, Mozambique, Morocco, Rwanda, Burkina Faso, Angola, Bangladesh, Papua New Guinea, Chile, Dominican Republic

Mathematics Textbook Development

Japan

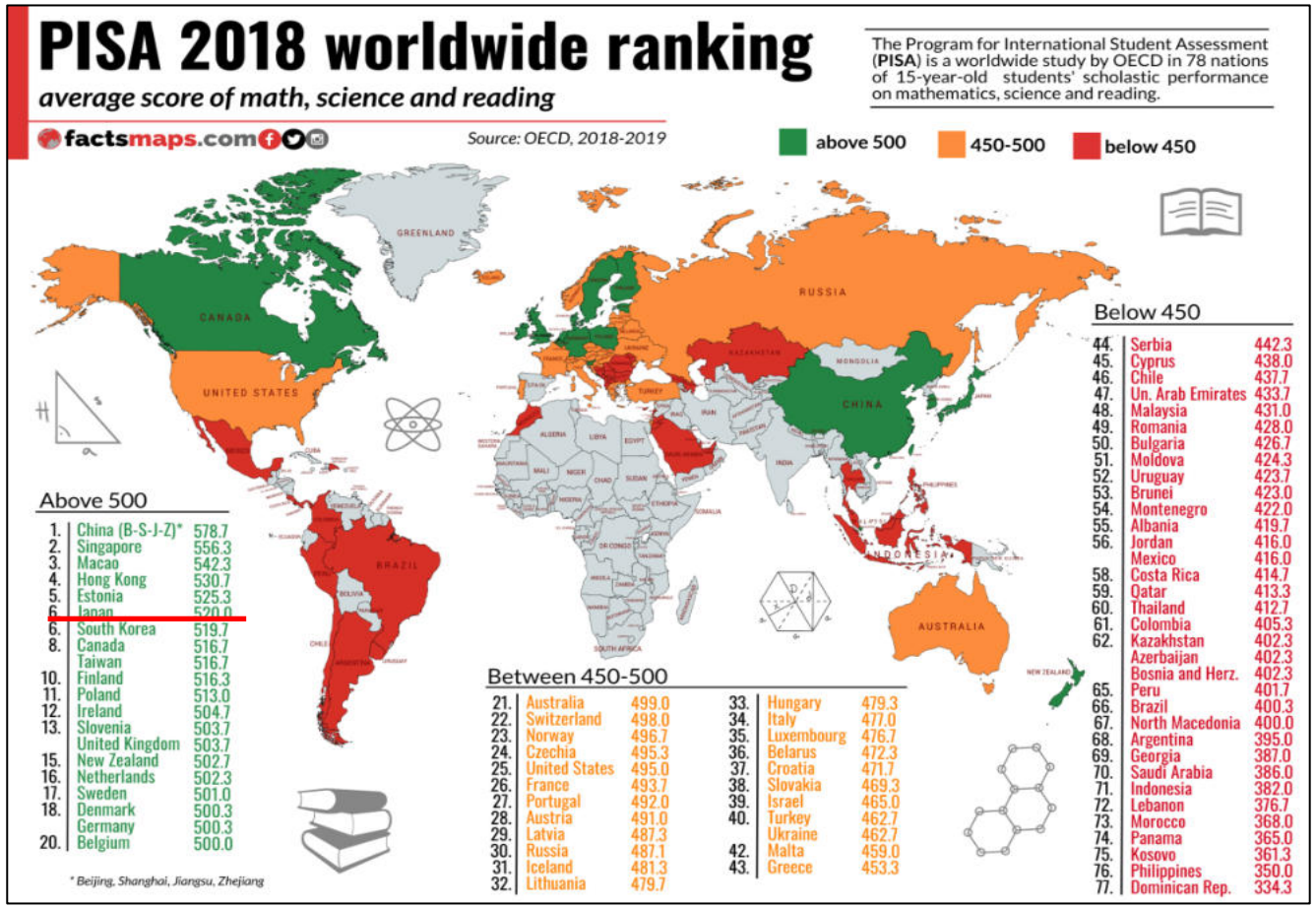
- Author, Elementary School Mathematics Textbooks (Gakko Tosho Co., Ltd.)
- Editor-in-Chief, English Edition of Elementary School Mathematics Textbooks (Gakko Tosho Co., Ltd.)

International Projects (JICA)

Mozambique, Papua New Guinea and Bangladesh

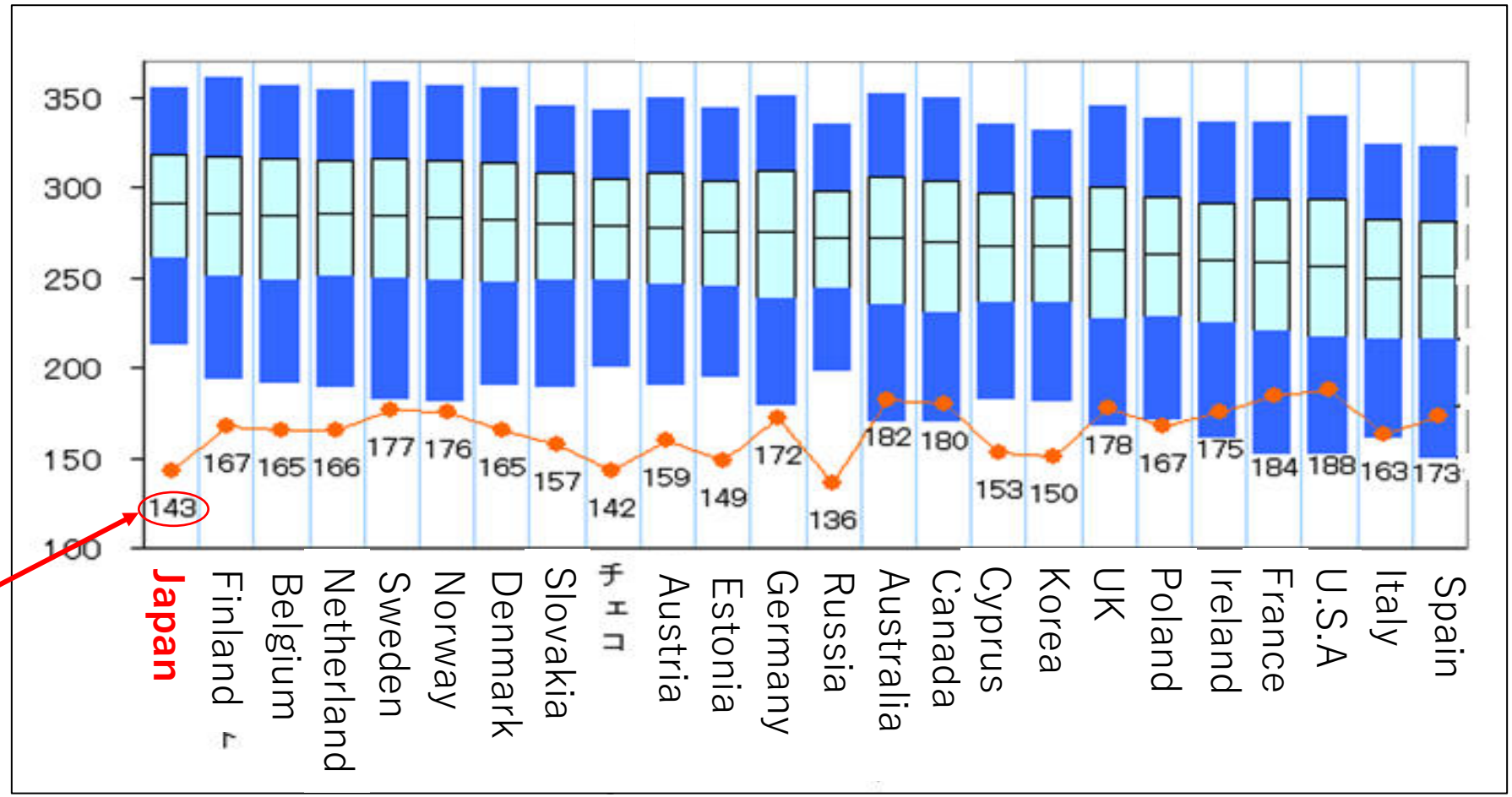
Before we begin・・・

The Program for International Student Assessment(PISA)



- ✓ The **Programme for International Student Assessment** (**PISA**), conducted every three years by the OECD, assesses the reading literacy, mathematical literacy, scientific literacy, and problem-solving skills of 15-year-old students.
  In 2018, **78 countries and economies** participated.
- ✓ **Japan consistently ranks among the top performers in all assessed domains in every cycle.**

Mathematical thinking skill



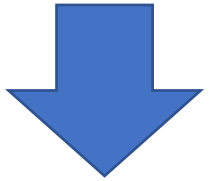
Difference in scores between those in the top 95% and those in the bottom 5%.

Source:OECD(2011)

Japan has a small gap in academic achievement between top and bottom students Especially, there are few bottom students.

Japan, not only **a small group of high-achieving students**, but also raises **the overall level of achievement across the entire student**, resulting in relatively small learning gaps.

Why？



“Structural Understanding of Numbers”,

“Learning Consolidation”, and “Textbook Design”

What I would like to think about together today

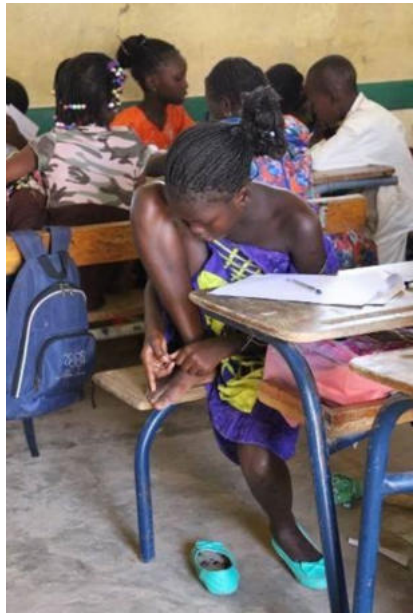
- What kind of understanding of numbers is needed so that **not only a small group of high-achieving students**, but **all students**, can smoothly understand and master the learning content? What does it mean to ensure the consolidation of learning?
- What kind of textbook structure enables **not only a small group of high-achieving students**, but **all students**, to smoothly understand and master the learning content?

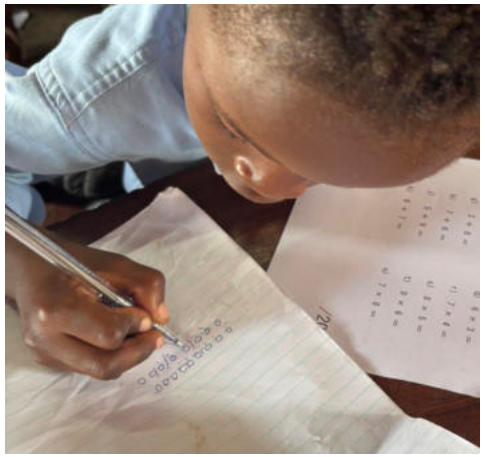
Today's my talk

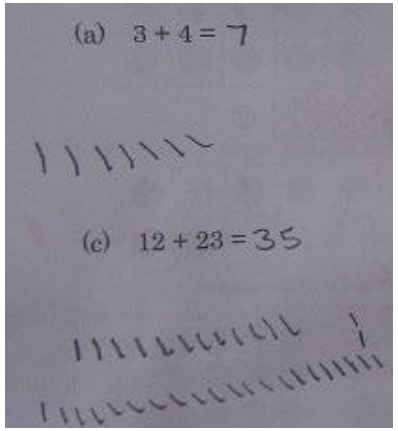
1. Moving beyond counting (Understanding numbers as structured units)
2. The importance of consolidating learning content
3. Textbook structure and the presentation of tasks that make (1) and (2) possible

1. Moving Beyond Counting

(Understanding numbers as groups / as units)







1. Moving Beyond Counting

(Understanding numbers as groups / as units)

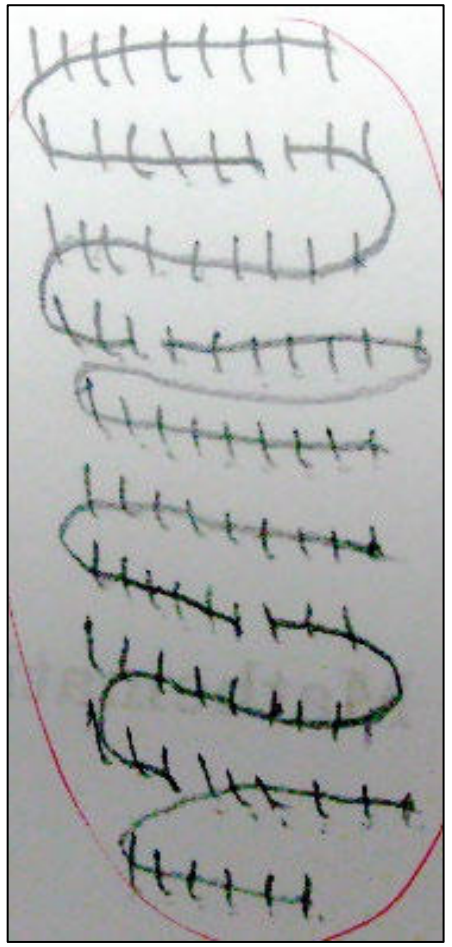
Introduction to a JICA Project Study

“Analysis of Mathematics Challenges in Zambian Primary Education”

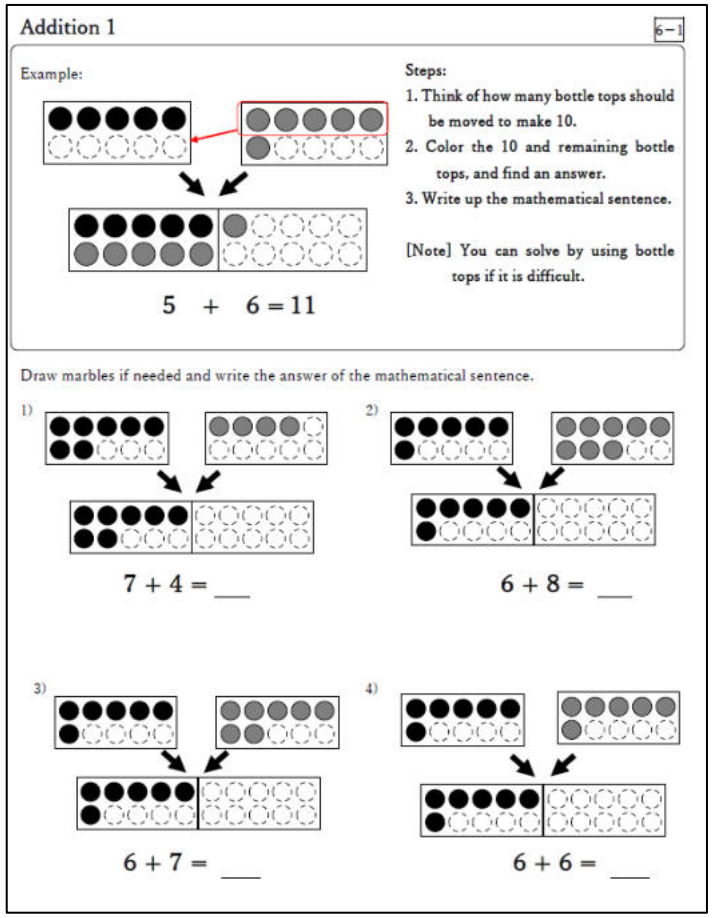
Our key question:

Why do children always count numbers?

- ✓ This child understands the **meaning of division**. However, the child does not perceive the number **95** as being composed of **nine tens and five ones**.
- ✓ As a result, the child interprets 95 as a number made up of **95 individual ones** and counts one by one all the way up to 95. In other words, the child lacks (or is unable to use) **structural thinking** about numbers.
- ✓ For children who understand numbers in this way, **calculation becomes a time-consuming and cognitively demanding task**.



Student Workbook Development (Finalized through repeated trials)



Addition 1

6−1

Example:

Steps:
1. Think of how many bottle tops should be moved to make 10.
2. Color the 10 and remaining bottle tops, and find an answer.
3. Write up the mathematical sentence.

[Note] You can solve by using bottle tops if it is difficult.

5 + 6 = 11

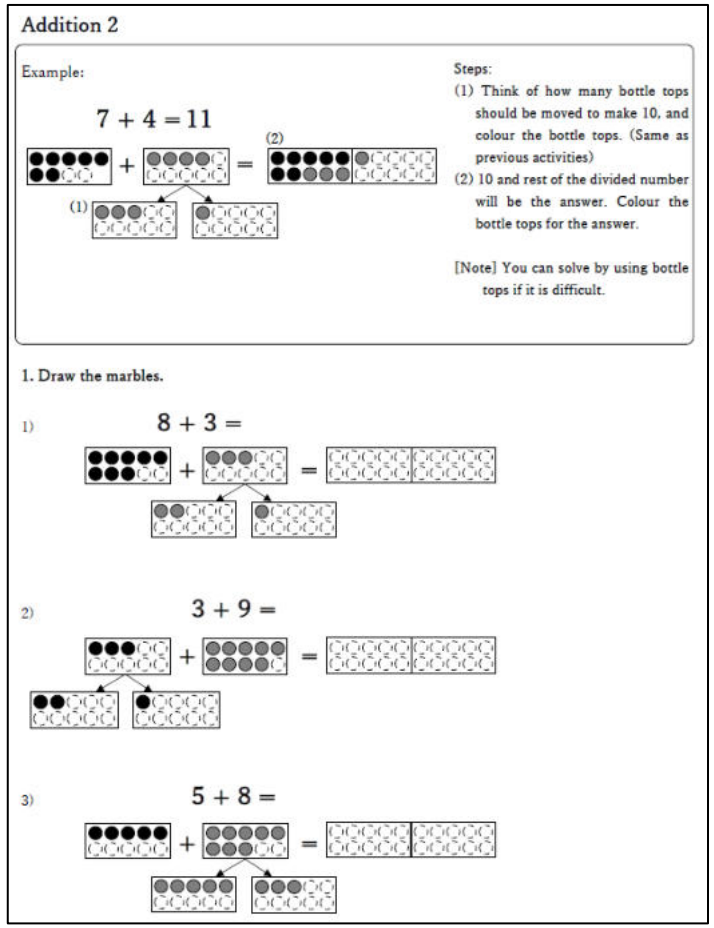
Draw marbles if needed and write the answer of the mathematical sentence.

1) 7 + 4 = ___

2) 6 + 8 = ___

3) 6 + 7 = ___

4) 6 + 6 = ___



Addition 2

Example:

7 + 4 = 11

(1) (2)

Steps:
(1) Think of how many bottle tops should be moved to make 10, and colour the bottle tops. (Same as previous activities)
(2) 10 and rest of the divided number will be the answer. Colour the bottle tops for the answer.

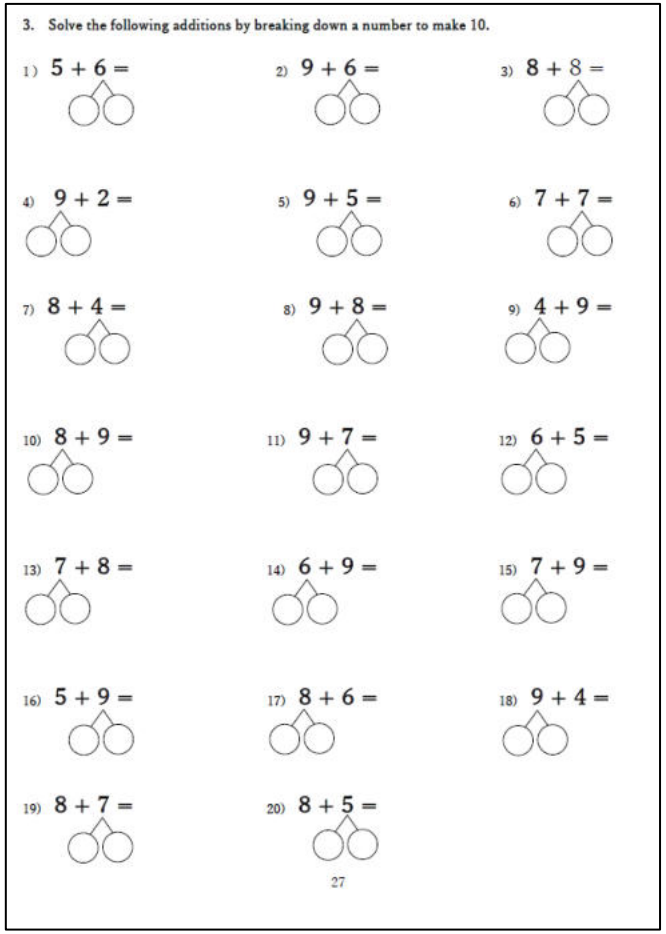
[Note] You can solve by using bottle tops if it is difficult.

1. Draw the marbles.

1) 8 + 3 =

2) 3 + 9 =

3) 5 + 8 =



3. Solve the following additions by breaking down a number to make 10.

1) 5 + 6 =
2) 9 + 6 =
3) 8 + 8 =
4) 9 + 2 =
5) 9 + 5 =
6) 7 + 7 =
7) 8 + 4 =
8) 9 + 8 =
9) 4 + 9 =
10) 8 + 9 =
11) 9 + 7 =
12) 6 + 5 =
13) 7 + 8 =
14) 6 + 9 =
15) 7 + 9 =
16) 5 + 9 =
17) 8 + 6 =
18) 9 + 4 =
19) 8 + 7 =
20) 8 + 5 =

27

結果

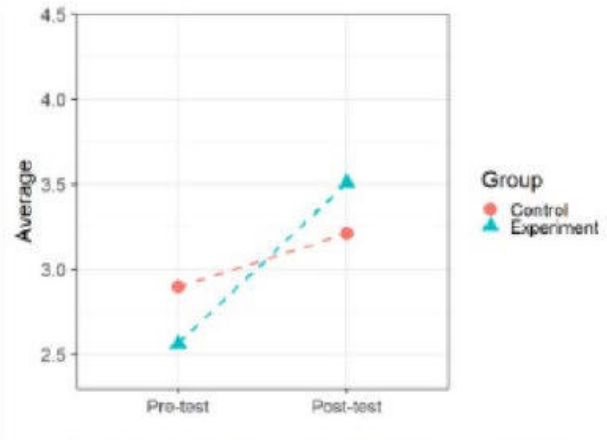


Figure 5　Result of grade 1 (11 tasks, N=20)

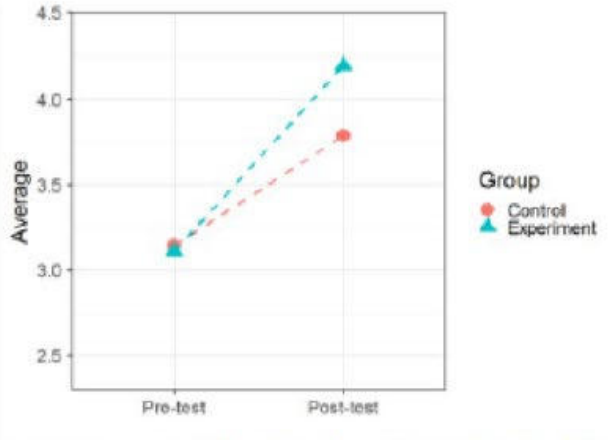


Figure 6　Result of grade 2 (9 tasks, N=20)

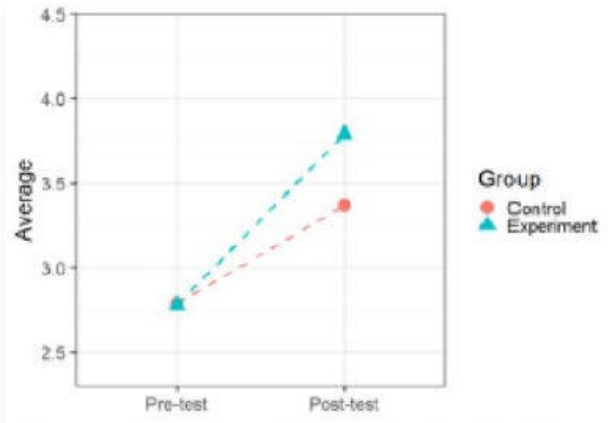


Figure 7　Result of grade 3 (20 tasks, N=20)

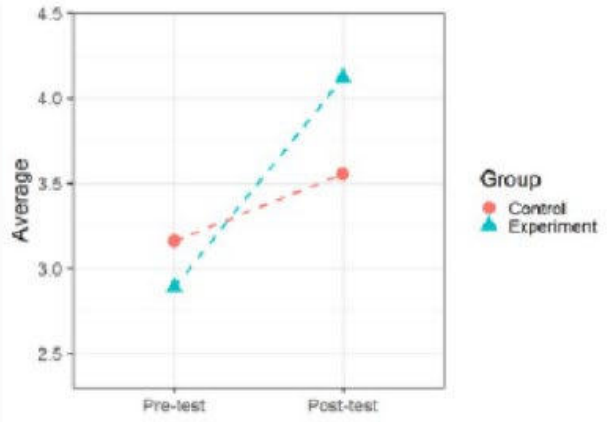
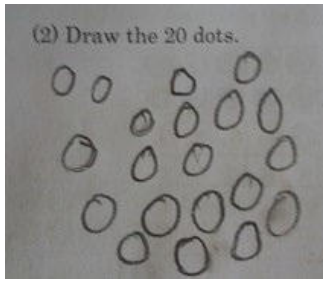
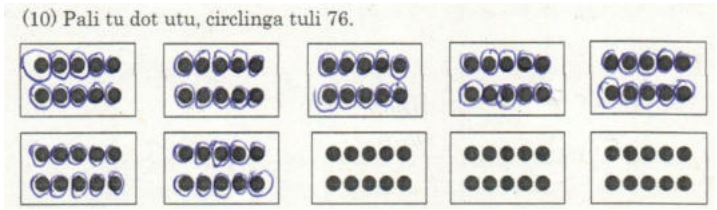
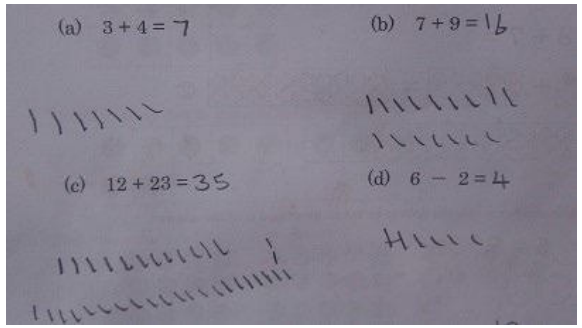


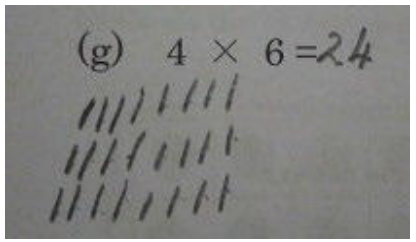
Figure 8　Result of grade 4 (20 tasks, N=20)

Before intervention

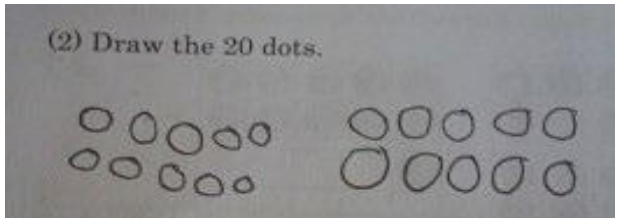


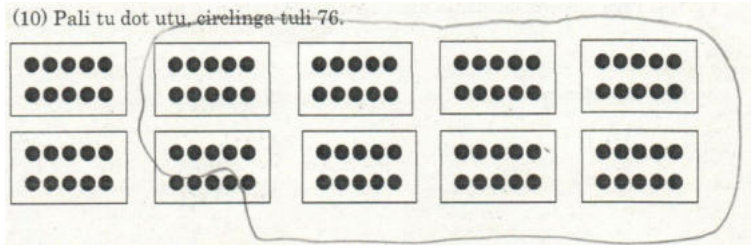


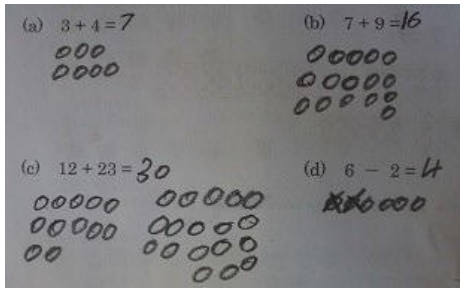


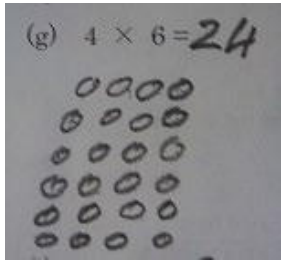


After intervention









Why do children always count numbers?

Our answer：

Because children are taught (and often encouraged or even forced) by teachers to rely on counting in all kinds of mathematical activities, including calculation.

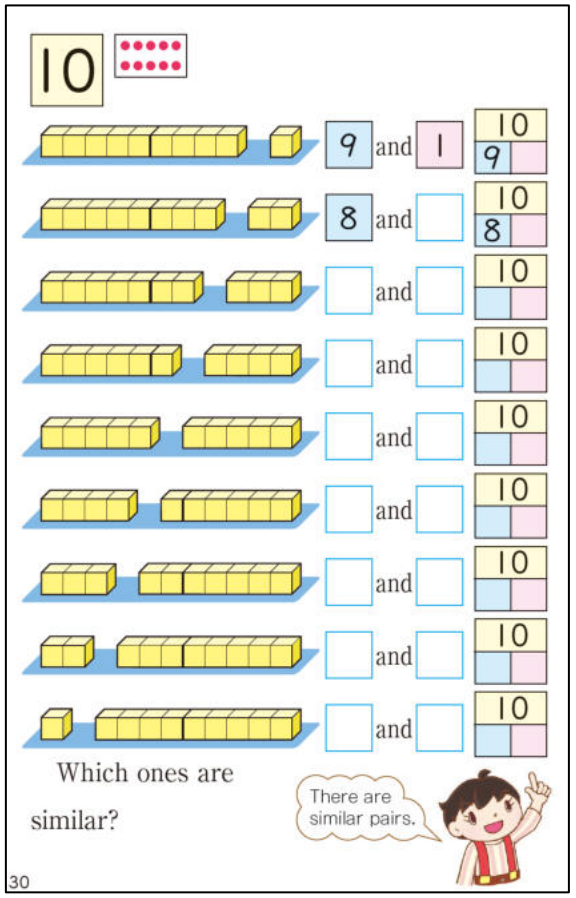
The ability to understand numbers as **structures (or groups)** can be **developed through instruction**. In other words, it is **neither an innate human ability nor a skill that is automatically embedded in culture**.

Therefore, **teachers need to explicitly address and teach this ability**.

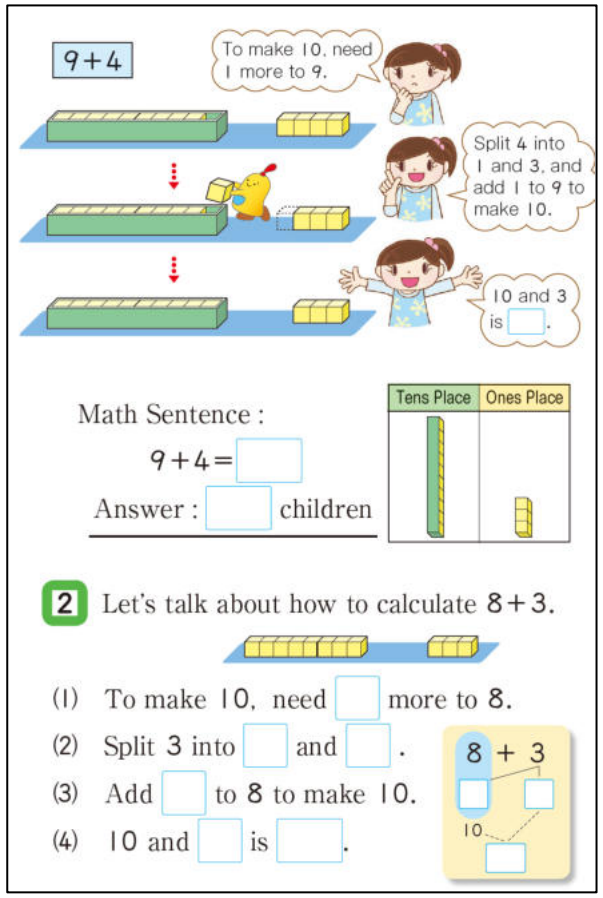
It is important to understand numbers as structures, not just to count them.

Reference: Japanese Grade 1 Mathematics Textbooks

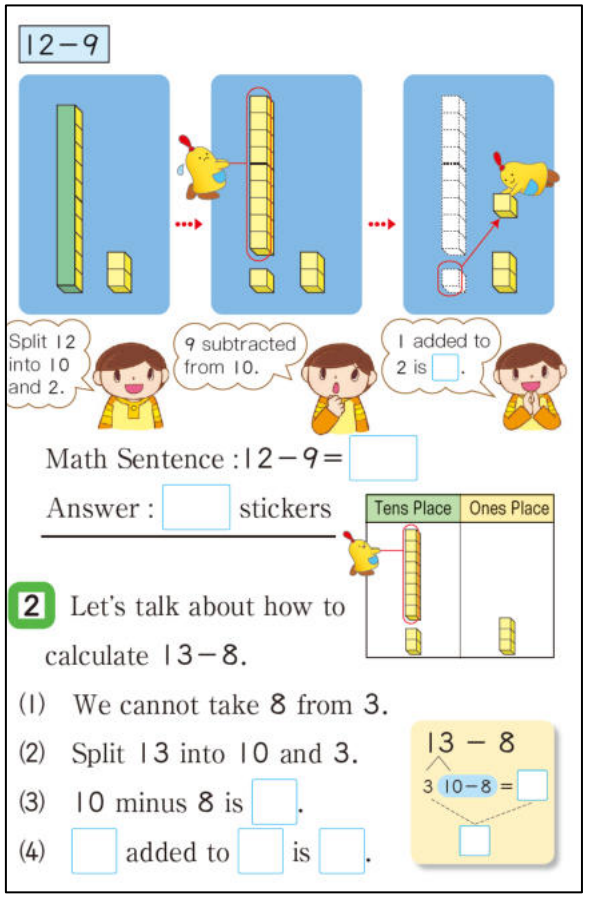
(Similar content will be included in the mathematics textbooks currently under development for Mozambique.)

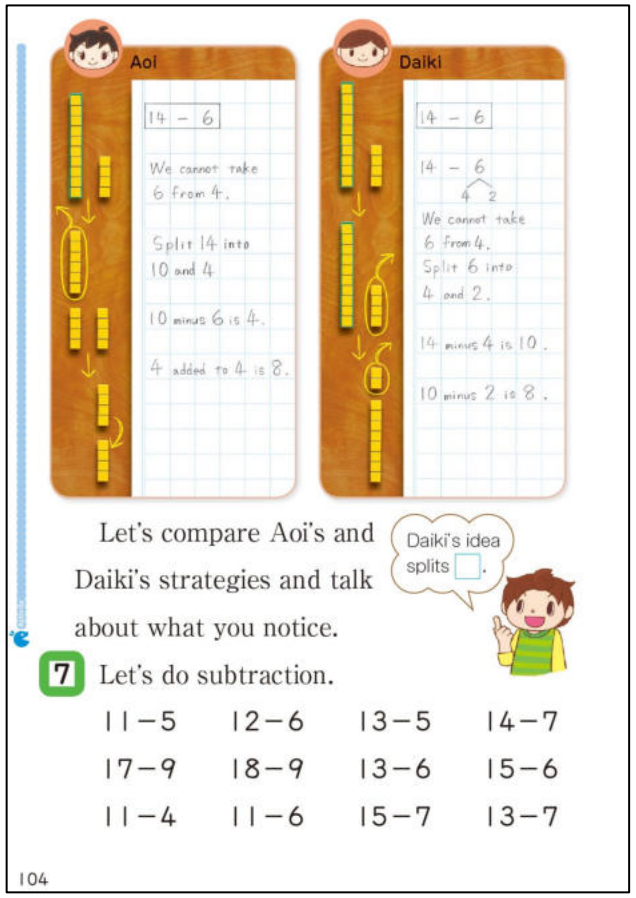


Composing and Decomposing 10



Addition of Single-Digit Numbers





Subtraction as the Inverse of Single-Digit Addition

2. The importance of consolidating learning content

Addition

1-digit ＋ 1-digit → Addition of more than 2-digit numbers

1. Addition of digits up to 10

0+0	1+0	2+0	3+0	4+0	5+0	6+0	7+0	8+0	9+0
0+1	1+1	2+1	3+1	4+1	5+1	6+1	7+1	8+1	9+1
0+2	1+2	2+2	3+2	4+2	5+2	6+2	7+2	8+2	
0+3	1+3	2+3	3+3	4+3	5+3	6+3	7+3		
0+4	1+4	2+4	3+4	4+4	5+4	6+4			
0+5	1+5	2+5	3+5	4+5	5+5				
0+6	1+6	2+6	3+6	4+6					
0+7	1+7	2+7	3+7						
0+8	1+8	2+8							
0+9	1+9								

2. Addition of digits up to 18

								9+2
							8+3	9+3
						7+4	8+4	9+4
					6+5	7+5	8+5	9+5
				5+6	6+6	7+6	8+6	9+6
			4+7	5+7	6+7	7+7	8+7	9+7
		3+8	4+8	5+8	6+8	7+8	8+8	9+8
	2+9	3+9	4+9	5+9	6+9	7+9	8+9	9+9

It is possible to add all numbers using the addition of a 1-digit number + another 1-digit number.

```
  42
+ 29
----
  71
```

1) 2 ＋ 9=11
2) (1+4)+2 = 7

```
 5358
+2762
-----
 8120
```

1) 8+2 = 10
2) (1+5)+6 = 12
3) (1+3)+7 = 11
4) (1+5)+2= 8

```
 5.83
+4.16
-----
 9.99
```

1) 3+6 = 9
2) 8+1 = 9
3) 5+4 = 9

Substraction

Subtraction of the inverse of an addition of two numbers to 1 number (up to 18)

→

Subtraction of numbers greater than or equal to 2

1. Subtraction of digits up to 10

0-0	1-0	2-0	3-0	4-0	5-0	6-0	7-0	8-0	9-0	10-1
	1-1	2-1	3-1	4-1	5-1	6-1	7-1	8-1	9-1	10-2
		2-2	3-2	4-2	5-2	6-2	7-2	8-2	9-2	10-3
			3-3	4-3	5-3	6-3	7-3	8-3	9-3	10-4
				4-4	5-4	6-4	7-4	8-4	9-4	10-5
					5-5	6-5	7-5	8-5	9-5	10-6
						6-6	7-6	8-6	9-6	10-7
							7-7	8-7	9-7	10-8
								8-8	9-8	10-9
									9-9	

2. Subtraction of digits up to 18

10-1	11-2	12-3	13-4	14-5	15-6	16-7	17-8	18-9
10-2	11-3	12-4	13-5	14-6	15-7	16-8	17-9	
10-3	11-4	12-5	13-6	14-7	15-8	16-9		
10-4	11-5	12-6	13-7	14-8	15-9			
10-5	11-6	12-7	13-8	14-9				
10-6	11-7	12-8	13-9					
10-7	11-8	12-9						
10-8	11-9							
10-9								

It is possible to do subtractraction of any numbers using subtraction up to 18

$$\begin{array}{r} 44 \\ -\,28 \\ \hline 16 \end{array}$$

1) $14 - 8 = 6$
2) $3 - 1 = 2$

$$\begin{array}{r} 5358 \\ -\,2762 \\ \hline 2596 \end{array}$$

1) $8 - 2 = 6$
2) $11 - 6 = 5$
3) $12 - 7 = 5$
4) $4 - 2 = 4$

$$\begin{array}{r} 6.15 \\ -\ 4.2 \\ \hline 2.95 \end{array}$$

1) $5\text{-}0 = 5$
2) $11\text{-}2 = 9$
3) $5\text{-}4 = 1$

Multiplication

1-digit × 1-digit → **Multiplication of more than 2-digit numbers**

It is possible to do multiplication of any numbers using 1-digit ×1-digit

1×1	2×1	3×1	4×1	5×1	6×1	6×1	7×1	8×1	9×1
1×2	2×2	3×2	4×2	5×2	6×2	6×2	7×2	8×2	9×2
1×3	2×3	3×3	4×3	5×3	6×3	6×3	7×3	8×3	9×3
1×4	2×4	3×4	4×4	5×4	6×4	6×4	7×4	8×4	9×4
1×5	2×5	3×5	4×5	5×5	6×5	6×5	7×5	8×5	9×5
1×6	2×6	3×6	4×6	5×6	6×6	6×6	7×6	8×6	9×6
1×7	2×7	3×7	4×7	5×7	6×7	6×7	7×7	8×7	9×7
1×8	2×8	3×8	4×8	5×8	6×8	6×8	7×8	8×8	9×8
1×9	2×9	3×9	4×9	5×9	6×9	6×9	7×9	8×9	9×9

$$\begin{array}{r} 46 \\ \times\ 8 \\ \hline 368 \end{array}$$

1) 8 × 6 = 48
2) 8 × 4 = 32

$$\begin{array}{r} 158 \\ \times\ 35 \\ \hline \end{array}$$

1) 5×8 = 40
2) 5×5 = 25
3) 5×1 = 5
4) 3×8 = 24
5) 3×5 = 15
6) 3×1 = 3

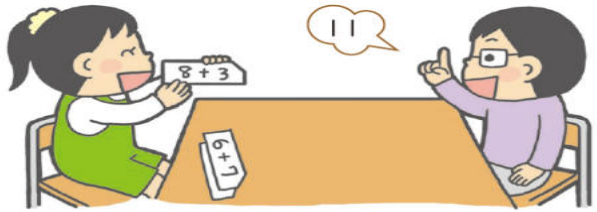
$$\begin{array}{r} 3.17 \\ \times\ 1.2 \\ \hline 2.95 \end{array}$$

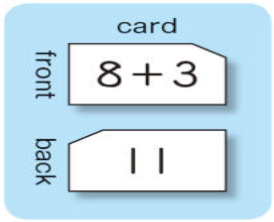
1) 2×7 = 14
2) 2×1 = 2
3) 2×3 = 6
4) 1×7 = 7
5) 1×1 = 11
6) 1×3 = 3

Exercises to acquire fluency of basic addition

11 Let's make addition cards and use them for practice.

❶ Say the answer.



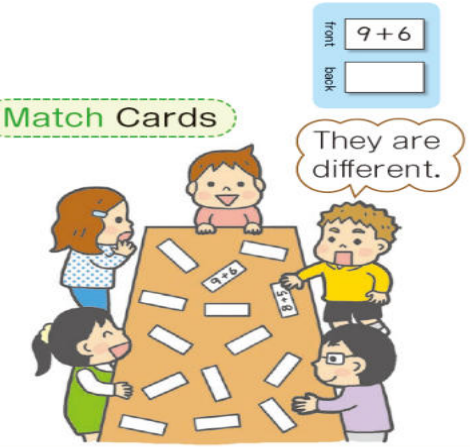


12 They lined up addition cards.

9+2	8+3	7+4		5+6
9+3		7+5	6+6	5+7
9+4	8+5	7+6	6+7	5+8
9+5	8+6		6+8	5+9
	8+7	7+8	6+9	
9+7	8+8	7+9		
9+8	8+9			
9+9				

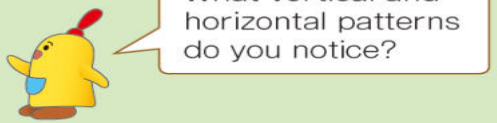
Let's talk about what you noticed.

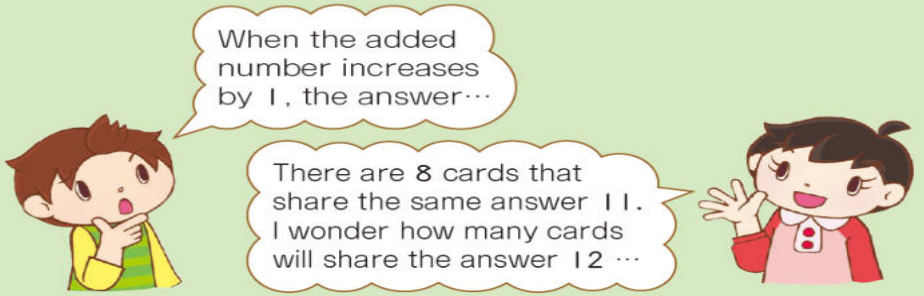
❷ Let's play a game



Write an expression on the blank card.

4+7	3+8	2+9
4+8		
4+9		

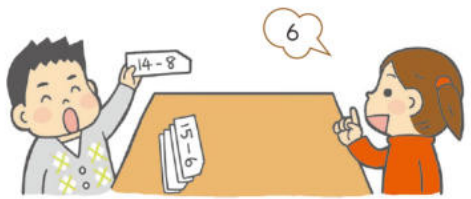




Exercises to acquire fluency of basic addition

11 Let's make subtraction cards and use them for practice.

❶ Say the answer.



card
front 14−8
back 6

12 They lined up subtraction cards.

11−2	12−3	13−4	14−5	15−6
11−3		13−5	14−6	15−7
11−4	12−5	13−6	14−7	15−8
11−5	12−6	13−7		15−9
11−6	12−7		14−9	
	12−8	13−9		
11−8	12−9			
11−9				

Let's talk about what you noticed.

❷ Let's play a game.

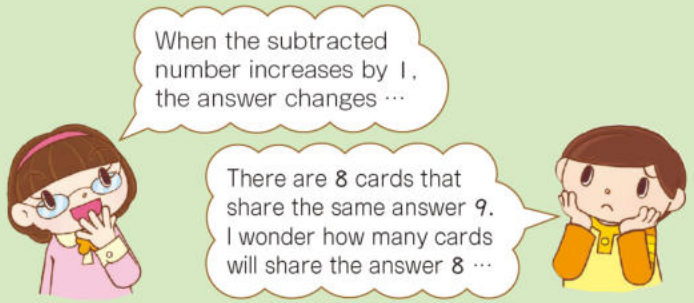
front 18−9
back

Pick up Cards 5

Match Cards

Write an expression on the blank card.

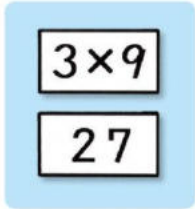
16−7	17−8	
16−8	17−9	
16−9		



Exercises to master multiplication table

Flash Card Games

Write multiplication expressions and answers for the multiplication table of 2,3,4 and 5 on each card to play the card game.



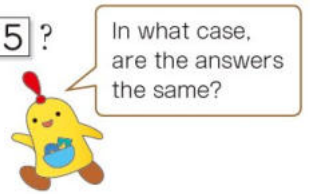
❶ Pick up answers.

❷ Matching cards (1)

❸ Comparing cards

Which is larger, 2×8 or 4×5 ?

How about 5×6 or 3×9 ?

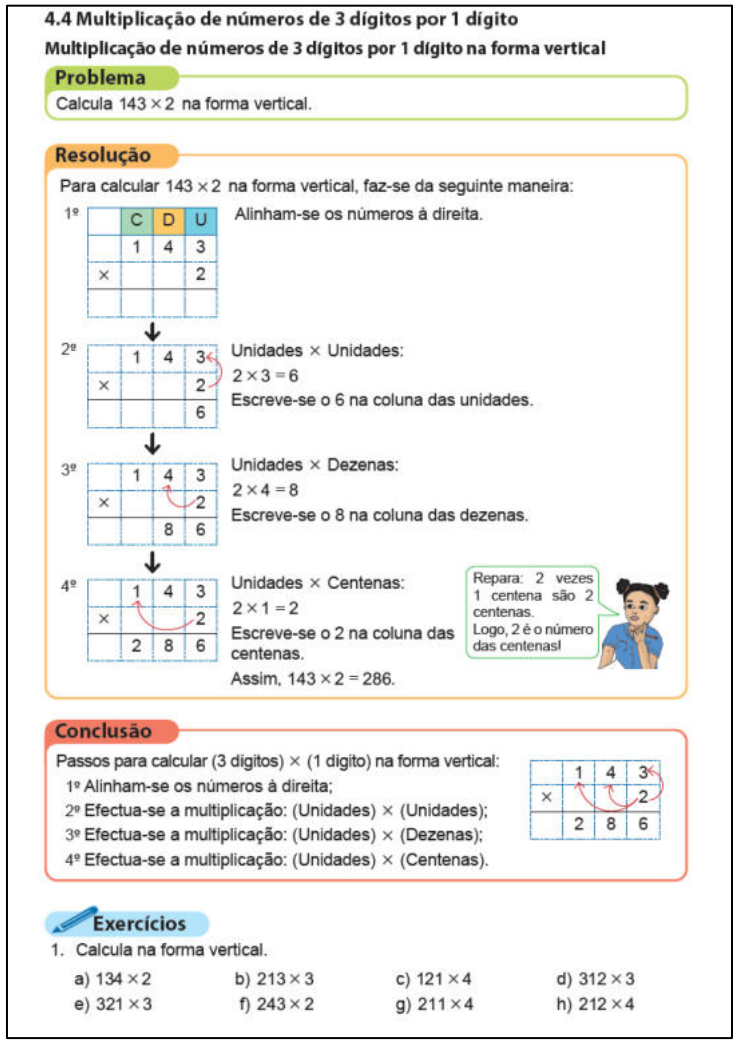


❹ Pick up answers (2)

If you pick up the 24 and 4×6 cards, can you keep them?

3. Structure of the Textbooks Being Developed under PRECEP

All lessons are structured following the same flow



4.4 Multiplicação de números de 3 dígitos por 1 dígito

Multiplicação de números de 3 dígitos por 1 dígito na forma vertical

Problema

Calcula 143 × 2 na forma vertical.

Resolução

Para calcular 143 × 2 na forma vertical, faz-se da seguinte maneira:

1º Alinham-se os números à direita.

2º Unidades × Unidades:
2 × 3 = 6
Escreve-se o 6 na coluna das unidades.

3º Unidades × Dezenas:
2 × 4 = 8
Escreve-se o 8 na coluna das dezenas.

4º Unidades × Centenas:
2 × 1 = 2
Escreve-se o 2 na coluna das centenas.

Repara: 2 vezes 1 centena são 2 centenas. Logo, 2 é o número das centenas!

Assim, 143 × 2 = 286.

Conclusão

Passos para calcular (3 dígitos) × (1 dígito) na forma vertical:
1º Alinham-se os números à direita;
2º Efectua-se a multiplicação: (Unidades) × (Unidades);
3º Efectua-se a multiplicação: (Unidades) × (Dezenas);
4º Efectua-se a multiplicação: (Unidades) × (Centenas).

Exercícios

1. Calcula na forma vertical.

a) 134 × 2 b) 213 × 3 c) 121 × 4 d) 312 × 3
e) 321 × 3 f) 243 × 2 g) 211 × 4 h) 212 × 4

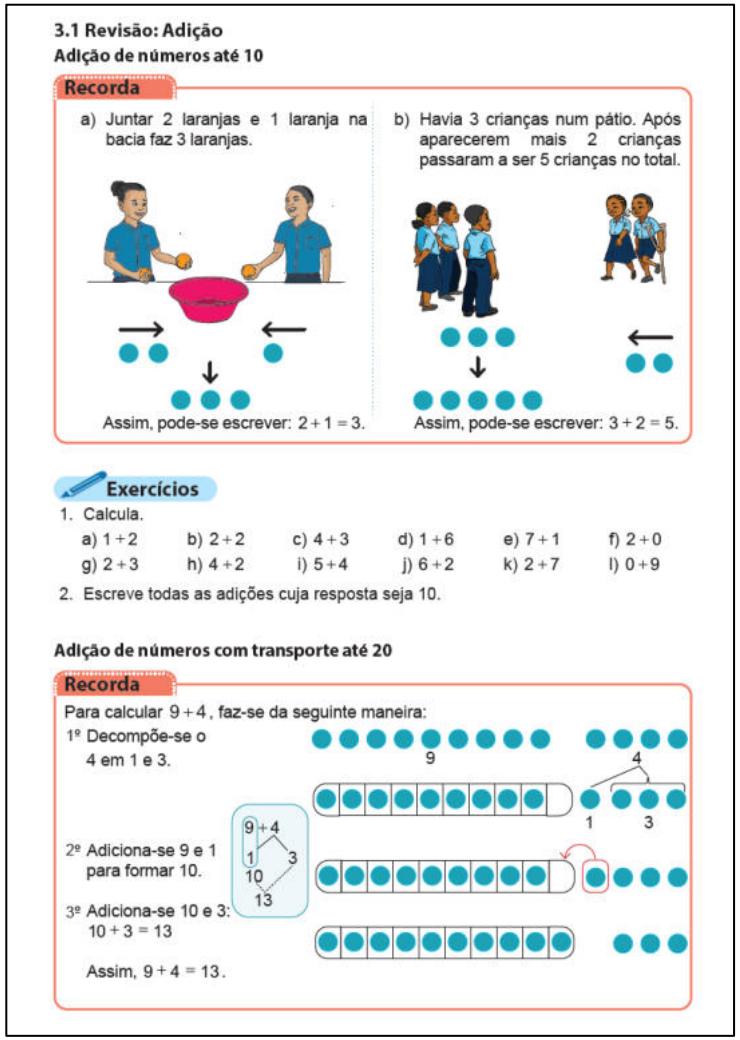
1) **"Problem": Presenting a new task (problem situation)**
   A problem designed with attention to links to the previous lesson and the overall learning progression (curriculum sequence).

2) **"Solution": Solving using learners' prior knowledge**
   Careful, step-by-step explanations.

3) **"Summary": Summarizing what has been learned**
   A recap of key new content (formulas, terms, methods, etc.).

4) **"Practice": Practice exercises for today's lesson**
   Providing sufficient volume of practice to support mastery and retention.

A review page designed to build readiness for learning new content



3.1 Revisão: Adição

Adição de números até 10

Recorda

a) Juntar 2 laranjas e 1 laranja na bacia faz 3 laranjas.

Assim, pode-se escrever: 2 + 1 = 3.

b) Havia 3 crianças num pátio. Após aparecerem mais 2 crianças passaram a ser 5 crianças no total.

Assim, pode-se escrever: 3 + 2 = 5.

Exercícios

1. Calcula.

a) 1 + 2	b) 2 + 2	c) 4 + 3	d) 1 + 6	e) 7 + 1	f) 2 + 0
g) 2 + 3	h) 4 + 2	i) 5 + 4	j) 6 + 2	k) 2 + 7	l) 0 + 9

2. Escreve todas as adições cuja resposta seja 10.

Adição de números com transporte até 20

Recorda

Para calcular 9 + 4, faz-se da seguinte maneira:

1º Decompõe-se o 4 em 1 e 3.

2º Adiciona-se 9 e 1 para formar 10.

3º Adiciona-se 10 e 3: 10 + 3 = 13

Assim, 9 + 4 = 13.

- ✓ Because mathematics learning is **highly sequential**, it is difficult for students to understand new content unless they have sufficiently mastered what they have learned previously.
- ✓ Reviewing the necessary prior content **before introducing new topics** makes learning more effective.
- ✓ In the **“Review (Recorda)”** section, previously learned content is **organized and summarized**, helping students consolidate their understanding.

A practice page designed to consolidate learned content

End of each section

Exercícios de consolidação

1. Calcula na forma vertical.

a) 218 + 154	b) 236 + 191
c) 318 + 23	d) 234 + 271
e) 655 + 63	f) 369 + 540
g) 145 + 262	h) 54 + 383
i) 692 + 8	j) 219 + 72
k) 189 + 306	l) 93 + 315
m) 183 + 420	n) 45 + 338
o) 350 + 159	p) 521 + 80

2. Calcula na forma vertical.

a) 428 + 195	b) 394 + 228	c) 386 + 57	d) 205 + 198
e) 67 + 898	f) 189 + 289	g) 545 + 367	h) 608 + 96
i) 137 + 96	j) 499 + 308	k) 25 + 496	l) 301 + 299
m) 254 + 667	n) 407 + 193	o) 497 + 159	p) 97 + 203

3. Calcula na forma vertical.

a) 815 + 532	b) 546 + 615	c) 716 + 430	d) 546 + 582
e) 809 + 624	f) 841 + 785	g) 542 + 963	h) 209 + 830
i) 290 + 956	j) 137 + 953	k) 361 + 643	l) 364 + 863
m) 469 + 882	n) 936 + 495	o) 675 + 758	p) 69 + 935

4. Resolve.

a) A senhora Fátima vendeu 456 balões no dia 1 de Junho e 74 balões no dia 2 de Junho. Quantos balões a senhora Fátima vendeu nos dois dias?

b) Uma associação de criadores de gado caprino vendeu no 1º semestre do ano, 426 cabritos e, no 2º semestre do mesmo ano, 485 cabritos. Quantos cabritos vendeu a associação naquele ano?

c) O senhor Maurício pretende fazer o muro de vedação da sua casa. Ele comprou 864 blocos para levantar o muro e 752 blocos para a fundação. Quantos blocos comprou, no total, o senhor Maurício?

End of each unit

Vamos confirmar o que aprendemos na Unidade 3

1. Calcula na forma vertical.

a) 386 + 232	b) 913 + 485	c) 146 + 205	d) 1527 + 2461
e) 26 + 104	f) 183 + 398	g) 408 + 762	h) 1245 + 330
i) 394 + 921	j) 49 + 381	k) 356 + 9	l) 407 + 293
m) 469 + 882	n) 521 + 80	o) 135 + 945	p) 282 + 85
q) 268 + 115	r) 397 + 134	s) 323 + 47	t) 759 + 685
u) 8067 + 1632	v) 98 + 204	w) 207 + 930	x) 361 + 665

2. Calcula na forma vertical.

a) 756 − 347	b) 486 − 79	c) 709 − 165	d) 639 − 571
e) 342 − 260	f) 246 − 93	g) 784 − 27	h) 425 − 298
i) 814 − 515	j) 245 − 87	k) 210 − 74	l) 703 − 584
m) 900 − 817	n) 2468 − 1346	o) 8297 − 40	p) 839 − 469
q) 8609 − 207	r) 3957 − 1236	s) 318 − 253	t) 473 − 125

3. Resolve.

a) Um distribuidor de material de escritório recebeu 356 cadeiras de escritório em Janeiro de um ano, e 467 cadeiras de escritório em Fevereiro do mesmo ano. Qual é a diferença entre o número de cadeiras que ele recebeu nos dois meses?

b) Uma empresa de sabonetes produziu 876 unidades numa semana e distribuiu 593 unidades na mesma semana. Quantas unidades ainda há por distribuir?

4. Calcula usando a forma mais fácil.

a) 48 + 3 + 7	b) 61 + 5 + 5	c) 29 + 4 + 16	d) 73 + 18 + 2
e) 6 + 39 + 4	f) 9 + 57 + 1	g) 13 + 46 + 7	h) 22 + 65 + 18

$32 + \underline{4 + 6} = 32 + \underline{10} = 42$ Vamos fazer primeiro a adição para formar 10 que é fácil.

- ✓ At the end of each section, a lesson titled “Exercícios de consolidação” is provided. By working on these practice exercises, learners can review the content studied in that section and consolidate their understanding.
- ✓ Similarly, at the end of each unit, a lesson entitled “Vamos confirmar o que aprendemos na Unidade” is included. Through these exercises, learners review the entire unit and strengthen their overall understanding of the content.

Thank you very much for your opinion and questions

skusaka@naruto-u.ac.jp